An Inviscid Approximation for the Centerline Deformation History in Entry Flows

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Potential flow theory is demonstrated to give a good approximation for the centerline kinematics in creeping, viscoelastic flows in planar, abrupt contractions. A slight shifting along the flow direction of the dimensionless axial velocity profile and of the centerline extension rate profile is needed to bring the inviscid results into best agreement with previous viscoelastic simulations. This shift is relatively insensitive to fluid elasticity level, and it is attributed to entrance effects in the smaller channel. Potential flow predictions are then used to develop a simple, analytical expression for the maximum extension rate in a planar, abruptly converging flow. The approximate formula does not require the material properties of the fluid under investigation, and it gives results which are in favorable agreement with the literature on viscoelastic fluids.

Elongational kinematics play a significant role in converging flows, particularly in the spatial regions near the flow centerline where the flow is purely elongational and the stresses can reach high levels. An understanding of the kinematics in this region is necessary in order to optimize polymer processing operations which involve converging flows. Previous approximate analyses (Cogswell, 1972; Binding, 1988) do not allow a practical estimation of the centerline deformation history for abrupt entry flows of arbitrary contraction ratio. (At high Reynolds number, James (1991) circumvents the problem of nonhomogeneous extension rate by designing the shape of his die so that a uniform extension rate would be generated along the die centerline.)

This work seeks to set forth a practical guide for estimating the centerline deformation history for planar flows and is motivated by previous extensional flow experiments. Kramer and Meissner (1980) measured the centerline velocity profiles in abrupt contraction flows of a low-density poly(ethylene) melt. Remarkably, the normalized axial profiles of the velocity and extension rate were very similar, apart from a very small shift along the axial coordinate, despite a tenfold increase in the maximum centerline extension rate.

In the work of Macosko et al. (1982), planar stagnation flows of a poly(styrene) melt were generated in a symmetric, hyperbolic-shaped die with both no-slip and lubricated walls. The measured centerline deformation history and elongational stresses were remarkably similar in the both cases.

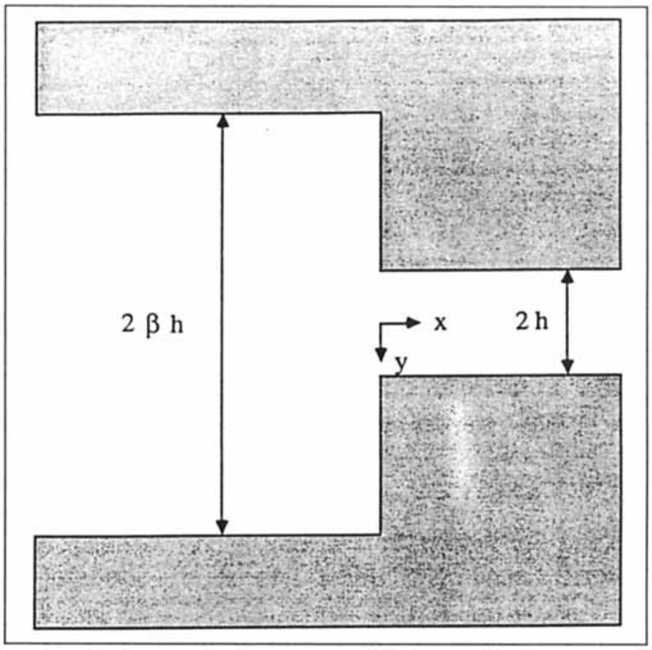

Figure 1. Schematic of planar entry flow.

The relevant Cartesian coordinate system is shown. The downstream channel half-width is h, and contraction ratio is β.

Finally, in the work of Mackley and Moore (1986), flow in a planar, wine-glass-shaped die was studied. The centerline velocity profiles were measured in flows of high density poly(ethylene) melts; and it was found that the profiles of the centerline velocity, normalized by the fully developed centerline velocity in the die land, were relatively *independent* of the polymer grade, the temperature, and the flow conditions. Flow geometry, rather than material properties, was the most important factor in establishing the centerline deformation history. This observation is somewhat curious in view of the fact that the centerline extension rates were well into the range in which published data show that nonlinear behavior of the

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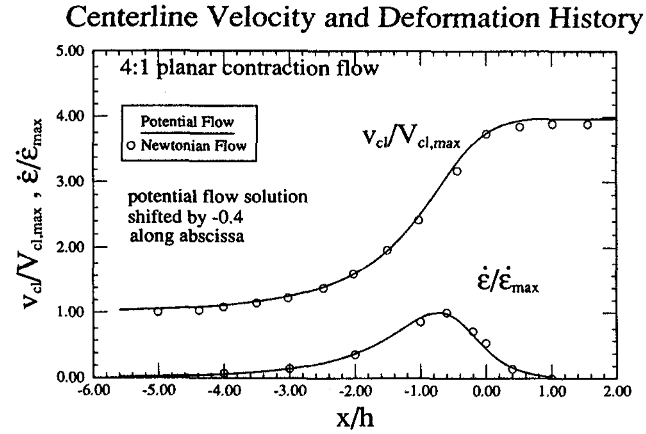

Figure 2. Comparison of nondimensional centerline velocity profiles and extension rates.

The data points have been read from plots set forth by Choi et al. (1988) for their finite difference simulations of the Newtonian flow problem. The contraction ratio is 4:1.

elongational viscosity would be expected (Han, 1976; Ide and White, 1978).

Extensional Deformation History in Abruptly Converging Flows

On the basis of the work discussed thus far, it would appear to be reasonable to neglect fluid memory, nonlinear effects, and the no-slip condition at the confining walls in an approximate analysis of the centerline kinematics in an abrupt entry flow. Also, due to the irrotational nature of the flow along the centerline as well as the disappearance of shearing stresses along this line, the inviscid flow solution would seem to provide a plausible, first approximation.

The two-dimensional contraction flow geometry with contraction ratio β is depicted in Figure 1 for the case of $\beta=4$. For abrupt contractions, from the treatment given by Milne-Thomson (1968), the inviscid centerline velocity profile is represented implicitly as (Galante, 1991):

$$\frac{v_{cl}}{V_{cl,\max}} \equiv s \tag{1a}$$

$$\left(\frac{x}{h}\right) = \frac{\beta}{\pi}\left\{\ln\left[\frac{s-1}{s+1}\right] + \frac{1}{\beta}\ln\left[\frac{\beta+s}{\beta-s}\right]\right\} \tag{1b}$$

Here $V_{cl,\max}$ is the plug-flow velocity in the larger channel far upstream of the entry plane. The local extension rate $\dot{\epsilon}(x)$ is given by

$$\dot{\epsilon}(x) = \frac{dv_x}{dx} = \frac{\pi V_{cl,\max}}{2\beta h}\left\{\frac{(s^2-1)(s^2-\beta^2)}{(1-\beta^2)}\right\};\ 1 \le s(x/h) \le \beta \tag{2}$$

Comparison of potential flow theory with creeping flow results

Equations 1 and 2 were derived for two-dimensional kinematics; and in the experimental entry flow studies discussed above, the flows did not satisfy this condition. Hence, these equations will be compared with numerically simulated results taken from the literature for the centerline kinematics of two-dimensional, abruptly converging flows at zero Reynolds number.

In the work of Choi et al. (1988), calculations were carried out over a range of Weissenberg numbers *We* for 4:1 planar contraction flows of "Leonov-like fluids"; and in Figure 2 the theoretical predictions of Eqs. 1 and 2 are compared with calculated results taken from Choi et al. (1988) for the Newtonian flow problem. On the whole, there is remarkably good agreement in the normalized axial profiles of the velocity and of the deformation rate, despite the dramatic differences between the governing equations and boundary conditions in the potential flow and viscous flow cases.

A small negative shift of $\Delta(x/h) = -0.4$, determined by inspection, has been applied along the flow direction in presenting the inviscid theory results. The need for this shift is related to an entrance effect in viscous flow in the smaller channel, and the magnitude is reasonably consistent with predictions of the hydrodynamic entrance length set forth by Boger (1982), albeit for circular entry flows. If we were concerned with a die that was symmetric about the entry plane, no shift would be required. For example, it is shown in Galante (1991) that for a two-dimensional flow in a smoothly converging, hyperbolic-shaped die, the normalized, centerline velocity profiles are identical for potential flow and creeping viscous flow.

To examine the range of validity of the inviscid flow approximation, one may consider Figure 16 of Choi et al. (1988), which gives results at higher *We*. Regardless of increases in elasticity level, for positions *upstream* of the entry plane, the normalized velocity profiles for all values of *We* are very similar, qualitatively and quantitatively. We contend that the agreement between the viscoelastic and the Newtonian results occurs because the upstream flow kinematics change slowly enough, relative to the timescale of the fluid, that memory and nonlinear effects do not play a significant role. In other words, the necessary conditions for the validity of Second-Order Fluid Theory are satisfied; and the Newtonian velocity profile should indeed hold by virtue of the Giesekus-Tanner theorem (Bird et al., 1987). However, for positions closer to and beyond the

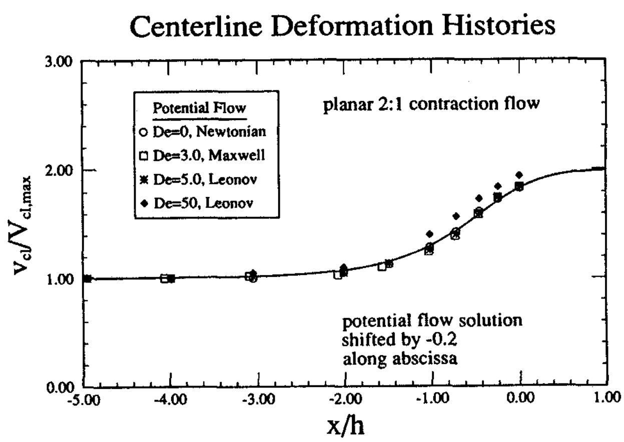

Figure 3. Nondimensional axial velocity profiles in a 2:1 contraction flow.

The data points have been taken directly from the work of Upadhyay and Isayev (1986). The fluid rheology under consideration is indicated in the legend. Note the agreement between the simulated profiles and that of potential flow. The agreement is quantitatively good at all Deborah numbers except for the highest.

Table 1. Comparison of Maximum Extension Rates for Potential Flow and Viscous Flow

Investigator	Fluid	β	*De* or *We*	$\dot{\epsilon}_{max}/\bar{\dot{\gamma}}_w$ Obs.	$\dot{\epsilon}_{max}/\bar{\dot{\gamma}}_w$ Eq. 4	Upstream Aspect Ratio
Kramer & Meissner (1980)	LDPE	10	unknown	0.177	0.194	1
	LDPE	10	~5× above	0.174	0.194	1
Xu et al. (1986)	Silicone Oil	12.2	0	0.165	0.195	1.1
	PDMS fluid	12.2	42	0.115	0.195	1.1
Upadhyay & Isayev (1986)	Newtonian	2	0	0.143	0.147	∞
	Maxwell	2	3	0.177	0.147	∞
	2-mode Leonov	2	5	0.167	0.147	∞
	2-mode Leonov	2	50	0.137	0.147	∞
Choi et al. (1988)	Newtonian	4	0	0.161	0.184	∞
	Leonov-like	4	1.5	0.147	0.184	∞

entry plane, the flow kinematics change drastically relative to the timescale of the fluid; hence, when viscoelastic flows are considered, the present potential-flow analysis is restricted in its validity to the upstream region. In the absence of fluid elasticity, however, Figure 2 would suggest that indeed there is a correspondence to potential flow theory well into the die land.

Further evidence that the potential solution provides a good approximation of the centerline kinematics in viscous flow is set forth in Figure 3. The points represent the calculations of Upadhyay and Isayev (1986) for flow in a planar, 2:1 contraction. The potential flow calculations have been empirically shifted, as in Figure 2. The size of the shift is smaller due to the lower contraction ratio β, since longer entrance lengths are expected in flows with higher β.

The shifted inviscid flow solution in Figure 3 compares well with the viscoelastic velocity profile for all but the highest elasticity level, $De=50$. Stronger fluid memory brings about larger entrance effects. However, even in the $De=50$ case, simply a different shift along the abscissa would significantly improve the agreement. The implication is that entrance effects are nearly independent of elasticity for low to moderate elasticity levels.

Estimating the maximum extension rate in abrupt, planar, converging flows

Our goal is to develop an approximation for the maximum extension rate $\dot{\epsilon}_{max}$ to which a fluid element is subjected in an abrupt, planar contraction flow at low Reynolds number. The contention is that the potential-flow, centerline kinematics can be utilized in estimating $\dot{\epsilon}_{max}$, regardless of the elasticity level. This maximum extension rate occurs somewhere in the "slowly varying" upstream region.

From Eq. 1, the maximum extension rate is found to be

$$\dot{\epsilon}_{max}=\frac{\pi}{8}\frac{V_{cl,max}}{\beta h}(\beta^2-1) \tag{3}$$

To relate this to readily measurable properties, $V_{cl,max}$ is first expressed in terms of the plug-flow, centerline velocity in the downstream channel, $v_{cl,max}$, where of course $v_{cl,max}=\beta V_{cl,max}$. Next, we propose that $v_{cl,max}$ be equated with the centerline velocity of a viscous, parabolic velocity profile for channel flow in a slit of the same size, $v_{cl,max}=\dot{\gamma}_w h/2$. Thus, the maximum extension rate can then be related to the wall shear rate in the die land $\dot{\gamma}_w$ through

$$\dot{\epsilon}_{max}=\pi\left(1-\frac{1}{\beta^2}\right)\frac{\dot{\gamma}_w}{16} \tag{4}$$

In assuming a parabolic velocity profile for the flow in the die land, shear-thinning has been neglected. If the fluid in question does shear-thin, we suggest in the usual manner that the *nominal wall shear rate* $\bar{\dot{\gamma}}$ be used in Eq. 4 in place of $\dot{\gamma}_w$, where $\bar{\dot{\gamma}}_w\equiv 3Q/(2h^2)$. Thus to get an approximation of the maximum extension rate, one only needs a simple measurement of the bulk flow rate per unit channel depth Q. On dimensional grounds, $\dot{\epsilon}_{max}$ should scale with Q. The novel feature of Eq. 4 is that the scaling coefficient is given in simple, analytical form as a function of β.

For values of the contraction ratio $\beta\geq 4$, Eq. 4 predicts that the maximum value of the local extension rate is about twenty percent of $\dot{\gamma}_w$; and the relation between the maximum extension rate and the downstream wall shear rate is relatively *independent* of the contraction ratio for $\beta\geq 4$. The latter observation is reminiscent of the tentative conclusion reached by Ramamurthy and McAdam (1980) that the axial velocity profile in an abruptly converging flow is independent of β for $\beta\geq 4$.

A comparison of Eq. 4 with the experimental data and simulated results from the studies cited previously is set forth in Table 1. Because of the similarity of the upstream velocity profiles in the work of Choi et al. (1988), only two cases from their work were considered. The table gives the contraction ratio β, the aspect ratio, the value of *De* or *We* (depending on which was cited), and the observed and predicted values of $\dot{\epsilon}_{max}/\bar{\dot{\gamma}}_w$, obtained by graphically differentiating the axial velocity profiles set forth in those studies.

There are few experimental results reported in the literature which satisfy all assumptions used in deriving Eq. 4. Note that in those cited in Table 1, the flow was not two-dimensional in the upstream channel. Also, in the studies of Xu et al. (1986), inertial effects were present to some degree. Further, in some simulations (Upadhyay and Isayev, 1986; Choi et al., 1988) recirculating vortices were observed in the upstream corners of the die. Despite these problems which are not taken into account in our potential flow development, the comparison of these literature results to Eq. 4 is quite good. Aside from the

PDMS work of Xu et al. (1986), Eq. 4 is in error in the worst case by only about twenty percent. The agreement between Eq. 4 and data from a wide variety of fluids is consistent with the conclusion, reached also by Mackley and Moore (1986), that flow geometry rather than material properties is the most important factor in the centerline extensional kinematics.

To summarize, in all cases set forth in Table 1, the simple form of Eq. 4 provides a reasonable engineering approximation for the maximum extension rate in a planar, abruptly converging flow without any prior knowledge of the properties of the fluid under investigation.

Summary

Motivated by published experimental data and numerical results, we empirically observe that inviscid flow theory can be used to give an approximation for the centerline kinematics in planar viscoelastic, converging flows at zero Reynolds number. Because of entrance effects, namely the development of the velocity profile in the smaller channel, a slight shifting along the flow direction is needed in order to bring about the best agreement between the nondimensional, axial velocity profiles for the polymeric and inviscid flows. We suspect the success of the potential flow solution is effected by the irrotationality of the flow along the centerline.

The potential flow solution was used in developing an analytical prediction for the maximum extension rate in a planar, abruptly converging die. This expression gave results which were in favorable agreement with experimental measurements and numerical simulations available in the literature and is useful in practical applications, since no knowledge of fluid rheology is required.

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